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The equation of a line is commonly written in either **gradient-intercept form** or in **general form**.

GRADIENT-INTERCEPT FORM

$y = mx + c$ is called the **gradient-intercept form** of an equation of a line.

The line with equation $y = mx + c$ has gradient m and y -intercept c .

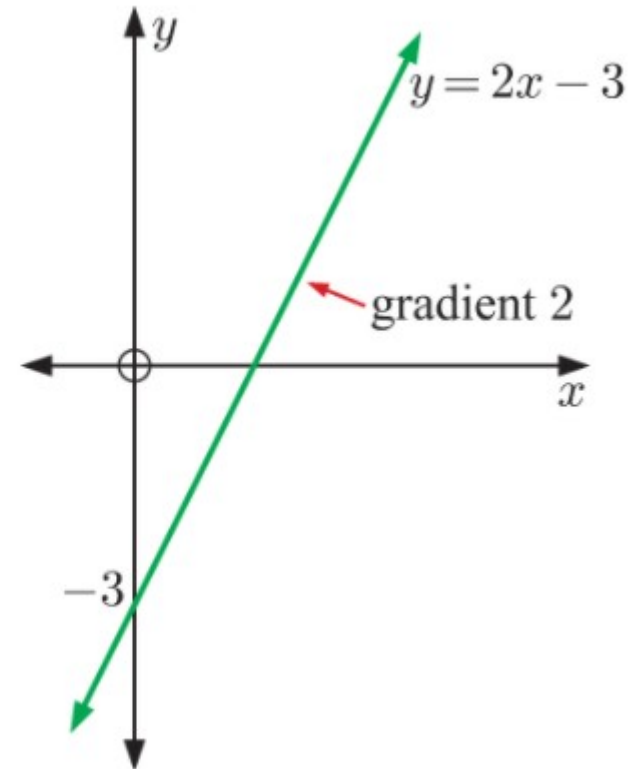
GRADIENT-INTERCEPT FORM

$y = mx + c$ is called the **gradient-intercept form** of an equation of a line.

The line with equation $y = mx + c$ has gradient m and y -intercept c .

For example, the line with equation $y = 2x - 3$ has gradient 2 and y -intercept -3 .

The **y -intercept** of a line is the y -coordinate of the point where the line cuts the y -axis.



GENERAL FORM

$Ax + By = C$ is called the **general form** of the equation of a line.

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For example, the equations $2x + 3y = 5$ and $x - 6y = -7$ are in general form.

Equations in general form are usually written with a positive coefficient of x .

FINDING THE EQUATION OF A LINE

If we are given enough information about a line, we can determine its equation.

To determine the equation of a line, we need to know either:

- its gradient and at least one point which lies on the line, or
- two points which lie on the line.

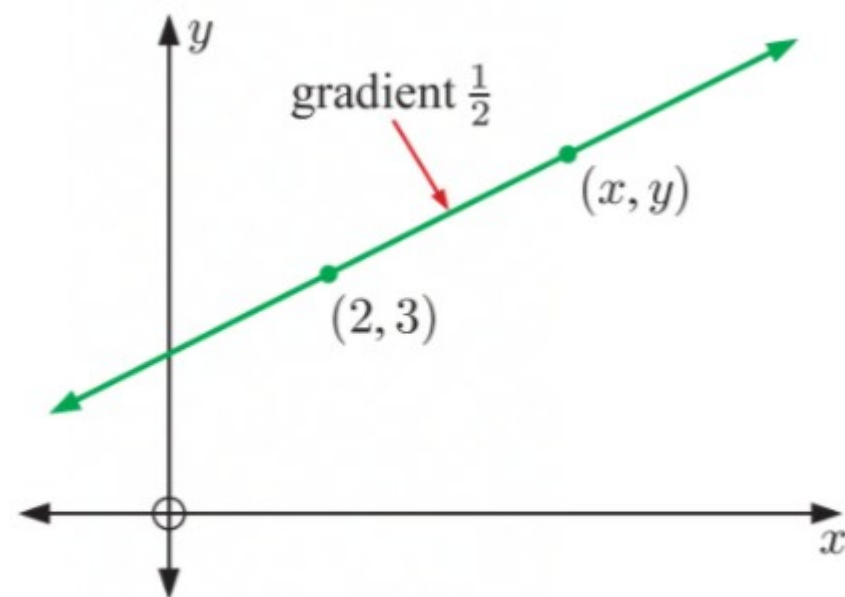
Suppose that a line has gradient $\frac{1}{2}$, and passes through the point $(2, 3)$.

For any point (x, y) which lies on the line, the gradient between $(2, 3)$ and (x, y) is $\frac{y-3}{x-2}$.

$\therefore$ the line has equation $\frac{y-3}{x-2} = \frac{1}{2}$

which can be written as $y-3 = \frac{1}{2}(x-2)$.

We can rearrange this to find the equation of the line in either gradient-intercept form or general form:

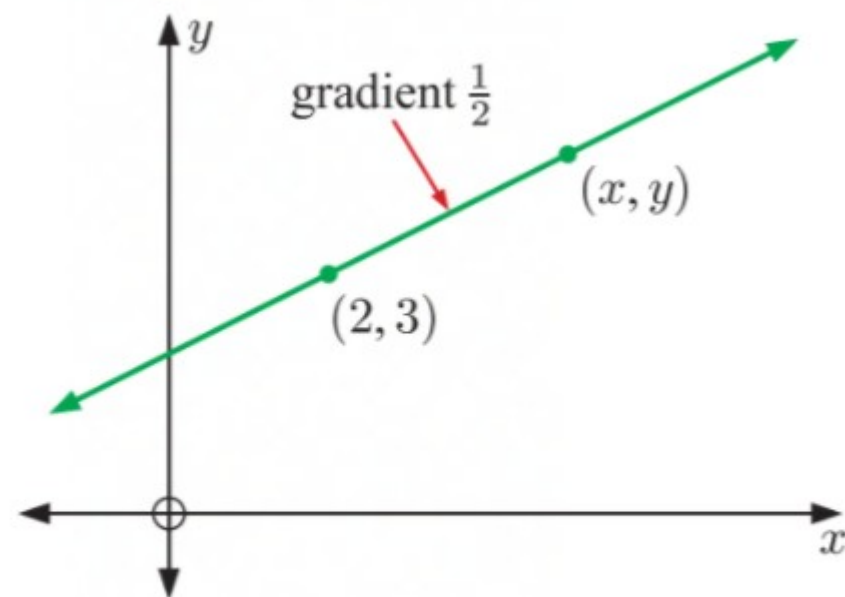


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Gradient-intercept form

$$y-3 = \frac{1}{2}(x-2)$$

$$\therefore y-3 = \frac{1}{2}x-1$$

$$\therefore y = \frac{1}{2}x+2$$

General form

$$y-3 = \frac{1}{2}(x-2)$$

$$\therefore 2(y-3) = 1(x-2)$$

$$\therefore 2y-6 = x-2$$

$$\therefore x-2y = -4$$

If a straight line has gradient m and passes through (a, b) , then it has equation

$$\frac{y - b}{x - a} = m \quad \text{or} \quad y - b = m(x - a).$$

We can rearrange the equation into either **gradient-intercept form** or **general form**.

Example 13



Find, in *gradient-intercept form*, the equation of the line with gradient 5 that passes through $(-1, 3)$.

Example 13

Self Tutor

Find, in *gradient-intercept form*, the equation of the line with gradient 5 that passes through $(-1, 3)$.

The equation of the line is $y - 3 = 5(x - -1)$
 $\therefore y - 3 = 5(x + 1)$
 $\therefore y - 3 = 5x + 5$
 $\therefore y = 5x + 8$

We are given the gradient and a point which lies on the line.



EXERCISE 6E.1

- 1 Find, in *gradient-intercept form*, the equation of the line with:
- a gradient 2, passing through $(1, 3)$
 - b gradient -1 , passing through $(-1, 2)$
 - c gradient $\frac{2}{3}$, passing through $(-3, 1)$
 - d gradient $-\frac{4}{5}$, passing through $(4, -2)$
 - e gradient $-\frac{3}{4}$, passing through $(6, -5)$.

Example 14

Find, in *general form*, the equation of the line with gradient $\frac{3}{4}$ that passes through $(5, -2)$.

Example 14

Find, in *general form*, the equation of the line with gradient $\frac{3}{4}$ that passes through $(5, -2)$.

The equation of the line is $y - -2 = \frac{3}{4}(x - 5)$

$$\therefore 4(y + 2) = 3(x - 5)$$

$$\therefore 4y + 8 = 3x - 15$$

$$\therefore 3x - 4y = 23$$

2 Find, in *general form*, the equation of the line with:

a gradient 4, passing through (3, 5)

c gradient $\frac{1}{3}$, passing through (1, 4)

e gradient $\frac{2}{7}$, passing through $(-5, -5)$.

b gradient $-\frac{3}{5}$, passing through $(-2, 1)$

d gradient $-\frac{3}{4}$, passing through (0, 6)

Example 15



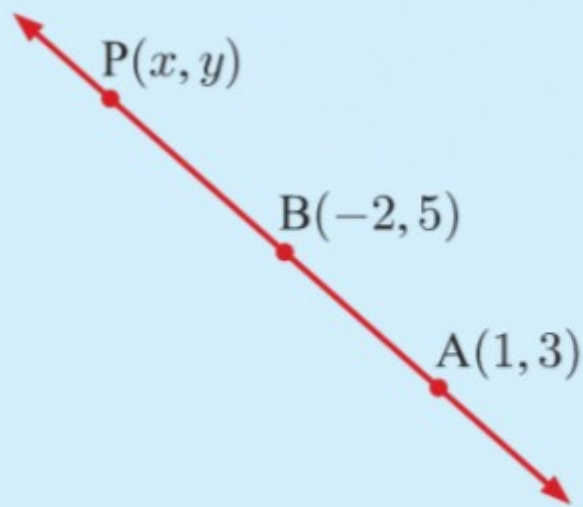
Self Tutor

Find, in *gradient-intercept form*, the equation of the line which passes through $A(1, 3)$ and $B(-2, 5)$.

Example 15



Find, in *gradient-intercept form*, the equation of the line which passes through A(1, 3) and B(−2, 5).



The line has gradient $= \frac{5 - 3}{-2 - 1} = \frac{2}{-3} = -\frac{2}{3}$,
and passes through the point A(1, 3).

$\therefore$ the equation of the line is

$$y - 3 = -\frac{2}{3}(x - 1)$$

$$\therefore y - 3 = -\frac{2}{3}x + \frac{2}{3}$$

$$\therefore y = -\frac{2}{3}x + \frac{11}{3}$$

We could use *either*
A or B as the point
which lies on the line.



3 Find, in *gradient-intercept form*, the equation of the line which passes through:

a $A(8, 4)$ and $B(5, 1)$

b $A(5, -1)$ and $B(4, 0)$

c $A(-2, 4)$ and $B(-3, -2)$

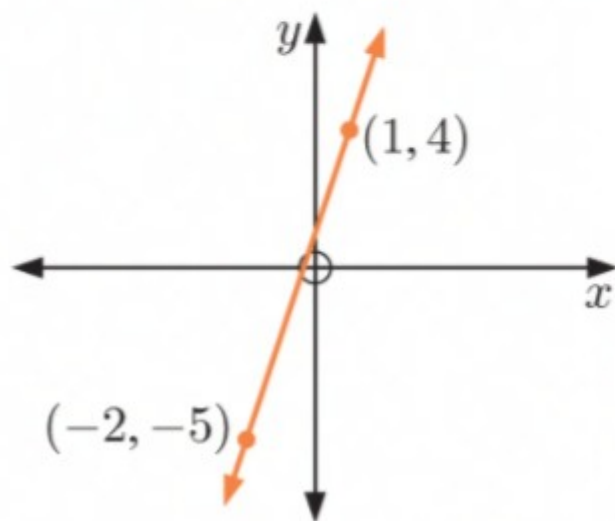
d $P(-4, 6)$ and $Q(2, 9)$

e $M(-1, -2)$ and $N(5, -4)$

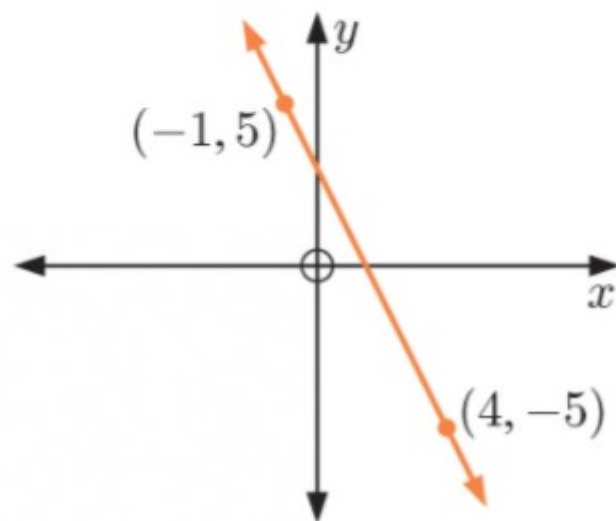
f $R(2, -4)$ and $S(7, -7)$.

4 Find, in *general form*, the equation of each of the following lines:

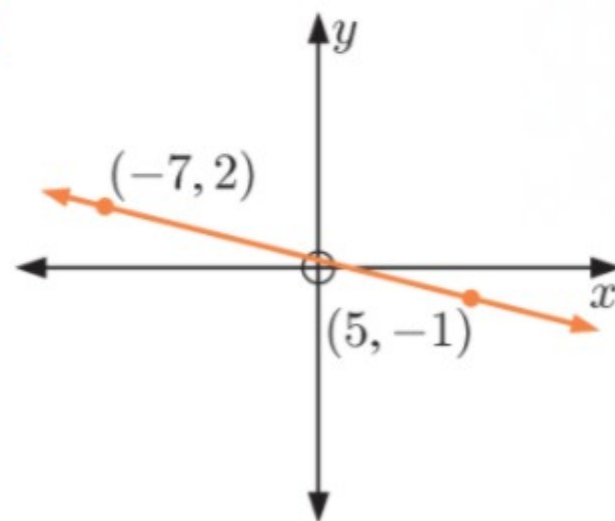
a



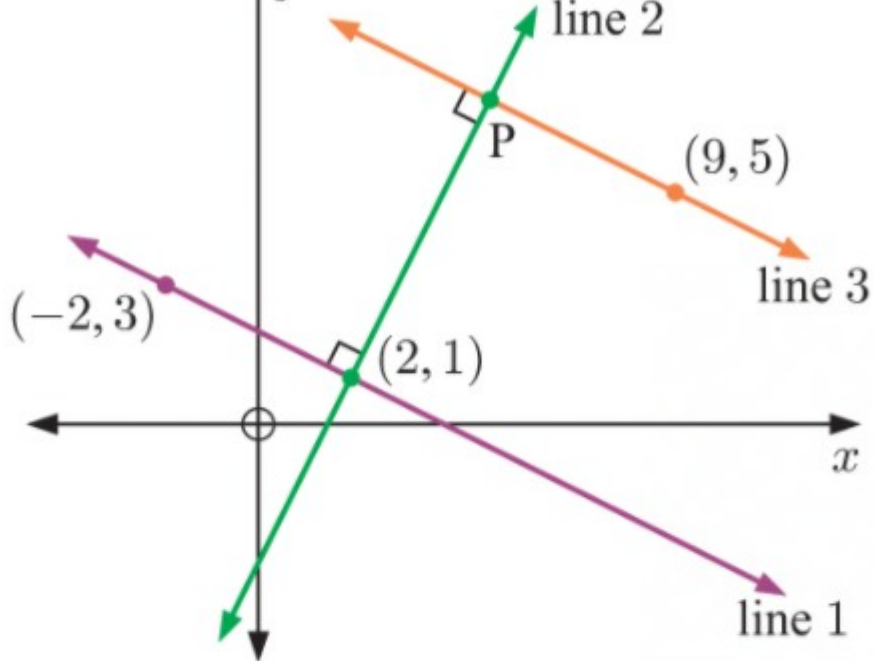
b



c

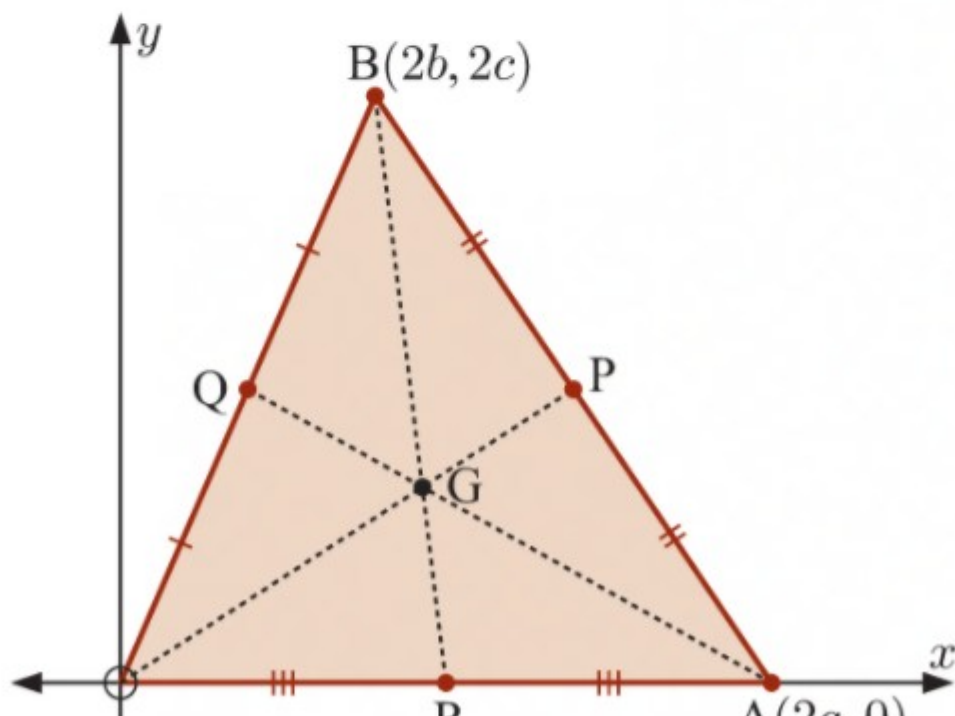


- 5** **a** Find, in general form, the equation of the line through $A(-3, 5)$ and $B(2, 1)$.
 b Show that the point $C(12, -7)$ also lies on this line.
- 6** Find the equation of the line which:
- a** cuts the x -axis at 5 and the y -axis at -2
 - b** cuts the x axis at -1 , and passes through $(-3, 4)$
 - c** is parallel to a line with gradient 2, and passes through the point $(-1, 4)$
 - d** is perpendicular to a line with gradient $\frac{3}{4}$, and cuts the x -axis at 5
 - e** is perpendicular to a line with gradient -2 , and passes through $(-2, 3)$.



- a** Find, in gradient-intercept form, the equation of:
- i** line 1 **ii** line 2 **iii** line 3.
- b** Show that the coordinates of P are (5, 7).

13



A **median** of a triangle is a line segment from a vertex to the midpoint of the opposite side.

- a** Show that [OP] has equation $cx - (a + b)y = 0$.
- b** Show that [AQ] has equation $cx - (b - 2a)y = 2ac$.
- c** Prove that the third median [BR] passes through the point of intersection G of medians [OP] and [AQ].

EXERCISE 6E.1

- | | | | |
|----------|--|--|---|
| 1 | a $y = 2x + 1$ | b $y = -x + 1$ | c $y = \frac{2}{3}x + 3$ |
| | d $y = -\frac{4}{5}x + \frac{6}{5}$ | e $y = -\frac{3}{4}x - \frac{1}{2}$ | |
| 2 | a $4x - y = 7$ | b $3x + 5y = -1$ | c $x - 3y = -11$ |
| | d $3x + 4y = 24$ | e $2x - 7y = 25$ | |
| 3 | a $y = x - 4$ | b $y = -x + 4$ | c $y = 6x + 16$ |
| | d $y = \frac{1}{2}x + 8$ | e $y = -\frac{1}{3}x - \frac{7}{3}$ | f $y = -\frac{3}{5}x - \frac{14}{5}$ |
| 4 | a $3x - y = -1$ | b $2x + y = 3$ | c $x + 4y = 1$ |
| | d $5x - 3y = 10$ | e $y = -3$ | f $x = 2$ |
| 5 | a $4x + 5y = 13$ | b $4(12) + 5(-7) = 48 - 35 = 13$ | ✓ |
| 6 | a $2x - 5y = 10$ | b $y = -2x - 2$ | c $y = 2x + 6$ |
| | d $4x + 3y = 20$ | e $x - 2y = -8$ | |

7 **a** $-\frac{3}{4}$ **b** $4x - 3y = 10$ **8** $7x - 5y = -42$

9 $y = \frac{3}{2}x - 3$ **10** **a** $y = \frac{2}{3}x - \frac{5}{3}$ **b** $-\frac{5}{3}$

11 $x + 3y = 13$

12 **a** **i** $y = -\frac{1}{2}x + 2$ **ii** $y = 2x - 3$ **iii** $y = -\frac{1}{2}x + \frac{19}{2}$

b **Hint:** Solve $2x - 3 = -\frac{1}{2}x + \frac{19}{2}$.

13 **Hint:** P is $(a + b, c)$, Q is (b, c)

a gradient of [OP] = $\frac{c}{a + b}$ etc.

b gradient of [AQ] = $\frac{c}{b - 2a}$ etc.

c Solve the equations in **a** and **b** simultaneously.